Teachers' Instrumentation of a Collaborative Dynamic Geometry Environment[1]

Muteb M. Alqahtani[*]
Arthur B. Powell[**]

Abstract

We draw on the theory of instrumental genesis (RABARDEL; BEGUIN, 2005) and the notion of co-action (HEGEDUS; MORENO-ARMELLA, 2010) to understand how teachers' instrumentation of dynamic geometry environment (DGE) and how this instrumentation shapes their geometric knowledge. In small groups, six middle and high school mathematics teachers engaged in solving open-ended geometric problems in an online dynamic geometry environment for 15 weeks. Our analysis of their interactions indicates that the co-action between the teachers and the environment helped them appropriate the dragging feature of DGE, which shaped their understanding of geometrical relations, particularly dependencies. Designing tasks that support teachers' effective appropriation of DGEs requires special attention to the co-active nature of DGEs. This study provides insights into aspects of learners' collaborative interaction with certain technologies.
Keywords: Dynamic Geometry, Teacher Knowledge, Technology, Professional Development.

Instrumentação de Professores em um Ambiente Colaborativo de Geometria Dinâmica

Resumo

Baseamo-nos na teoria da gênese instrumental (RABARDEL; BEGUIN, 2005) e a noção de co-ação (HEGEDUS; MORENO-ARMELLA, 2010) para entender como a instrumentação dos professores para o ambiente de geometria dinâmica (GD) e a forma como essa instrumentação molda seus conhecimentos geométricos. Em pequenos grupos, seis professores de matemática de ensino médio e universitário se dedicaram a resolver problemas geométricos abertos em um ambiente de geometria dinâmica on-line por 15 semanas. Nossa análise de suas interações indica que a coação entre os professores e o meio ambiente ajudou-os a se adequarem à característica de arrastar do AGD, que moldou sua compreensão das relações geométricas, em particular as dependências. O design de tarefas que apoiam a apropriação efetiva dos ambientes de GD dos professores exige atenção especial à natureza coativa dos ambientes de GD. Este estudo fornece informações sobre os aspectos da interação colaborativa dos alunos com certas tecnologias.
Palavras-chave: Geometria dinâmica, conhecimento do professor, tecnologia, desenvolvimento profissional.

Introduction

Understanding geometry is important in itself and for understanding other areas of mathematics. It contributes to one's logical and deductive reasoning about spatial objects and relationships. Geometry provides visual representations alongside the analytical representation of a mathematical concept (GOLDENBERG, 1988; LABORDE, 1999; PIEZ; VOXMAN, 1997). Pairing learning geometry with technological tools of Web 2.0 can allow learners to investigate collaboratively geometrical objects, properties, and relations and develop flexible understanding of geometry. The nature of mathematics entails collaborative settings for learning and teaching mathematics (GATTEGNO, 1987; PIMM, 1987) where learners have the opportunity to explore mathematics objects, notice their properties, and discuss relations among them. Integrating collaborative learning in classrooms is

recommended (NATIONAL GOVERNORS ASSOCIATION CENTER FOR BEST PRACTICES; COUNCIL OF CHIEF STATE SCHOOL OFFICERS, 2010) and a body of research outlines positive outcomes of collaboration in learning mathematics (see, for example, ONER, 2008; SPRINGER; STANNE; DONOVAN, 1999; STAHL, 2006; STAHL, 2015; WEBB; PALINCSAR, 1996). In particular, collaboration can support students' learning of different mathematical topics including dynamic geometry.

With dynamic geometry environments (DGEs), the software provides feedback to the user after manipulating dynamic objects, which affects the user's interaction with the software. The environment reacts to the users' actions through engineered infrastructure that responds according to the theory of geometry. This reaction can inform the users' actions and can shape users' thinking. Though teaching with technology is recommended (NATIONAL GOVERNORS ASSOCIATION

[*]Endereço eletrônico: muteb.alqahtani@cortland.edu
[**]Endereço eletrônico: powellab@newark.rutgers.edu

CENTER FOR BEST PRACTICES; COUNCIL OF CHIEF STATE SCHOOL OFFICERS, 2010, p. 7), meta-analytic and large-scale studies indicate that teaching with technology cannot guarantee positive learning outcomes (CHEUNG; SLAVIN, 2013; HIGGINS; XIAO; KATSIPATAKI, 2012; KAPUT; THOMPSON, 1994; LI; MA, 2010; RAKES; VALENTINE; MCGATHA; RONAU, 2010; WENGLINSKY, 1998). Careful investigations are required to understand the appropriation of technology and how it shapes mathematics learning, especially when it occurs in a collaborative setting. There is a gap in the literature about understanding how learners use technological tools collaboratively and develop their mathematical knowledge. To contribute to this understanding, we describe learners' appropriation of online, dynamic geometry environment and its influence on their geometrical understanding. This paper responds to the question: How do learners appropriate an online, collaborative dynamic geometry environment and how do this appropriation and the environment's co-active functionality shape their geometrical understanding?

Related Literature and Theoretical Perspective

The affordances of DGEs for mathematics learning have given rise to empirical and theoretical investigations. Powell and Grisi-Dicker (2012) observe that researchers have attended to three broad questions: (1) How learners interact with DGEs, (2) What subject matter learning occurs from interacting with DGEs, and (3) What types of interaction with DGEs support learning. Within these questions, some researchers have investigated how learners learn different geometric and algebraic topics of a DGE and how changes in their knowledge occur (FALCADE; LABORDE; MARIOTTI, 2007; HOHENWARTER; HOHENWARTER; LAVICZA, 2009). Guven, Cekmez, and Karatas (2010) studied how DGEs can support students' conjecturing and proving through explorations. Others studied cognitive processes linked to a defining feature of DGEs: dragging. Arzarello, Olivero, Paola, and Robutti (2002), Baccaglini-Frank and Mariotti (2010), and Hollebrands (2007) studied different type of dragging and use of dragging with trace and developed associated dragging categories and purposes. Talmon and Yerushalmy (2004) investigated the dragging and students understanding of dependency. Using the theory of

instrumental genesis, others have investigated how students transform technological tools into mathematical instruments (ALQAHTANI; POWELL, 2017; GUIN; TROUCHE, 1998; HEGEDUS; MORENO-ARMELLA, 2010; RABARDEL; BEGUIN, 2005; SAMPER; CAMARGO; MOLINA; PERRY, 2013). However, given that digital tools such as DGEs can be used in collaborative environments, work needs to be done to understand how collaborating with each other and digital tools learners shape their development of geometrical thinking. Since the defining feature of DGEs is the ability to drag objects and observe their behavior, it is important to invistigate how learners become aware of the dragging affordence of DGEs and use it to explore mathematical objects and relations.

To understand learners' appropriation of technological artefacts or tools, we draw on a Vygotskian perspective about goal-directed, instrument-mediated action and activity. Instrumental genesis (RABARDEL; BEGUIN, 2005) posits that users' (teachers', students', or, in general, learners') activity directed toward an object (material, mental, or semiotic) such as a task is mediated by tools, which may be material devices or semiotic constructs. To appropriate a tool, users develop their own knowledge of how to use it, a utilization scheme. This scheme together with the tool forms the instrument. Rabardel and Beguin (2005) emphasize that the instrument is not just the tool but rather is "a mixed entity, born of both the user and the object: the instrument is a composite entity made up of a tool component and a scheme component" (p. 442). Therefore, an instrument is a two-fold entity, part artefactual and part psychological.

The transformation of a tool into an instrument occurs through two dialectical processes that account for potential changes in the instrument and in the users, respectively, instrumentalization and instrumentation. In instrumentation, the structure and functionality of tools shape how learners use the tool, which result in shaping learners' thinking since it "concerns the emergence and development of utilization and instrumented action schemes" (RABARDEL; BEGUIN, 2005, p. 444). In instrumentalization, the learner's interactions with a tool also shapes the tool and how is used, "the learner enriches the artifact properties" (Rabardel & Beguin, 2005, p. 444). Particular infrastructural properties of DGEs give rise to a unique component of

instrumentation. Hegedus and Moreno-Armella (2010) theorize that DGEs capability of responding to users' movement of base points or hotspots establishes a dialectical co-active relationship. Hotspots are points that are "used to construct mathematical figures, e.g., join two points with a segment" (HEGEDUS; MORENO-ARMELLA, 2010, p. 26). As users drag (click, hold, and slide) a base point or hotspot of a geometric figure, the DGE redraws and updates information on the screen, preserving all constructed mathematical relations among objects of the figure. In redrawing, the DGE creates a family of not only visually but also mathematically similar figures. Users may attend to the reaction of the DGE and experience and understand underlying mathematical relations among objects such as dependencies.

The co-active infrastructural properties of DGEs and the process of instrumental genesis are theoretical considerations for implementing research on learners' appropriation of an online, collaborative dynamic geometry environment and how appropriation and the environment's co-active functionality shape learners' geometrical understanding. Theoretically, to support effective appropriation of the environment and awareness of its co-active features, we design sequences of tasks with specific characteristics (POWELL; ALQAHTANI, 2015). The sequence of tasks was structured to take advantage of the co-active nature of DGEs and to help learners transform the environment from an artifact into an instrument. The tasks invite learners to drag objects and notice their behavior, which encourages the learners to pay special attention to the hotspots of constructions as they drag them. Moreover, the tasks also permit learners to gain insights into the theory of geometry that guides objects' behavior within DGEs. The theories of instrumental genesis and co-action can likewise be used as analytical tools to inquire into how learners interact and appropriate DGEs as well as learn from their activities in the environment. Learners' understanding of hotspots, dragging, and dependencies relies in the co-active relation between the learner and the DGE, which also allows us to investigate the movement towards appropriating DGE and use it instrumentally.

Methods

This study is part of a larger project to investigate the development of significant mathematical discourse in the context of a cyberlearning environment that integrates digital tools for discussion and collaborative geometrical explorations—Virtual Math Teams with GeoGebra (VMTwG). VMTwG is a product of a collaborative research project among investigators at Rutgers University, Drexel University, and the Math Forum, which contains support for chat rooms with collaborative tools for mathematical explorations, including a multi-user, dynamic version of GeoGebra, where team members can define objects and drag hotspots around on their screens (see Figure 1). In this project, for 15 weeks, middle and high school teachers collaborative in teams in VMTwG to work on open-ended geometrical constructions and problems in the first half of the school year. In the second half of the year, they engage their students with learning geometry in VMTwG.
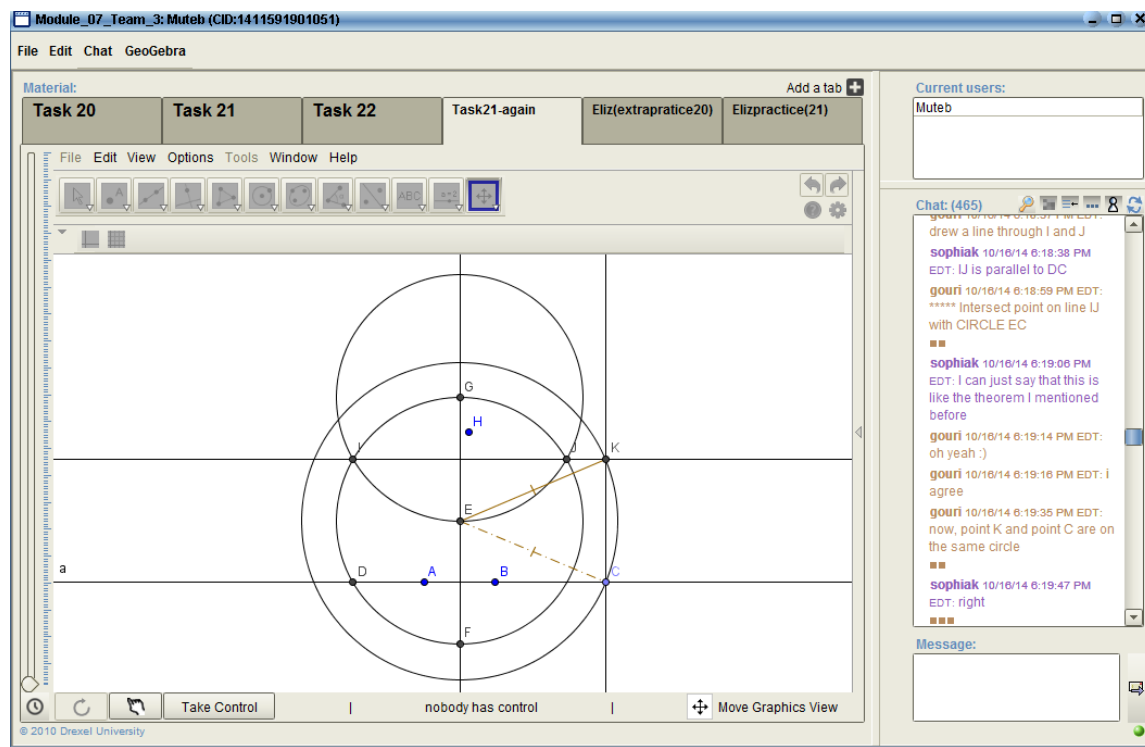
Data collection

For this study, data come from the professional development course, consisting of teachers' interactions in VMTwG while constructing geometric figures and solving open-ended geometrical problems. Using Vygotskian notions tool mediated activities and the importance of social interactions for learning, VMTwG is designed so that learners collaboratively manipulate geometrical objects in a shared dynamic geometry space (multiuser version of GeoGebra) and discuss properties of mathematical objects and relations among them in chat panel. The research team designed tasks for middle and high school teachers to work on in small teams. The tasks encourage the teachers to engage in a productive mathematical discourse that includes discussions about properties of mathematical objects and the relations among them (Powell & Alqahtani, 2015). The VMTwG environment records teachers' action in GeoGebra space and their chat postings. Each session is recorded in a video format and can be replayed using a replayer.

Four teams of teachers (13 teachers) participated in the professional development course in the first semester of 2013 and three teams (eight teachers) participated in the professional development course in the first semester of 2014. In each semester, teachers worked in small teams (2-4 teachers) for 15 weeks to solve geometrical tasks. In every week, teachers met online for two meetings in total of four hours. The first meeting in each week was usually designated to discussing geometrical

problems and the second meeting was for discussing readings and reviewing chat logs. However, some teams chose to work for more than three hours on the geometrical tasks in some weeks.

Figure 1: Team 3's construction of a perpendicular line that passes through an arbitrary point

Source: Authors' archive.

Data analysis

From the data collected in year 2013 and 2014, we focused on the work of Team 1 from 2013 and Team 3 from 2014. We selected these teams since they demonstrate conspicuously how teams attended to the environment's reaction to their actions. Team 1 from 2013 consisted of four middle school teachers and Team 3 from 2014 consisted of two high school teachers. Before this course, none of the six teachers had experience with dynamic geometry. The teachers met in VMTwG synchronously for two hours twice a week. We used the discursive and inscriptive data generated from their work on four different tasks. Team 1 worked to examine different types of triangles and construct them, then to re-examine the same triangles to discover dependencies involved in their construction (see Figure 2). In a later session in the course, Team 1 examined and then constructed perpendicular bisectors. Team 3 discussed the construction of equilateral triangle and then constructed one. They also constructed a perpendicular line that passes through an arbitrary point in a later session in the course.

Using conventional content analysis (Hsieh & Shannon, 2005), we analyzed data of interactions within each of the two teams to understand their instrumental genesis and its implications for their mathematical understanding. In our data analysis, we followed teachers' use of tools in VMTwG and the development of their utilization schemes as well as observed how teachers use these tools to solve geometrical tasks and discuss mathematical ideas that connect to the use of these tools. We also used the construct of co-action (HEGEDUS; MORENO-ARMELLA, 2010; MORENO-ARMELLA; HEGEDUS, 2009) to understand when, why, and how do teachers interact with hotspots (points used for construction); what feedback do they perceive and what they do with this feedback; and how does it shape their subsequent actions. We viewed changes in teachers' use of tools, such as identifying and dragging hotspots, as signs of teachers' development of utilization schemes that they employ to work on tasks. We also viewed changes

in teachers' responses to the environment's reactions as changes in their utilization schemes.

Results

Our analysis focuses on understanding how the teachers appropriated VMTwG and how this appropriation shapes their geometrical understanding. Our results show how through co-action teachers and VMTwG interact and how this interaction leads to shaping the teachers' understanding of affordances of dragging and the dynamic-geometry relation, dependency.

Teachers' appropriation of dragging was evident in teachers' work through their actions and tool use, which is an indicator of the development of teachers' usage schemes. We also found that teachers implemented their understanding of dependencies and dragging to solve subsequent geometrical problems. We will present the work of Team 1 in two different tasks. In the first task, Team 1 displayed an understanding of dependency in dynamic geometry and then implemented it in solving the second task, constructing perpendicular bisector. Similarly, we will present the work of Team 3 on other two tasks in VMTwG. The first task was concerning constructing equilateral triangle. Team 3 displayed an understanding of hotspots in this task and then implemented it while working on constructing a perpendicular line that passes through an arbitrary point. In the following sections, we describe the work of the two teams.

Team 1:

During the first collaborative session with simple constructions, Team 1's members quickly show some understanding of dependency in dynamic geometry. Team 1 worked to identify and construct different types of triangles and then to re-examine different triangles to discover dependencies involved in their construction. Figure 2 shows the different triangles that the teachers examined. The vertices of first triangle, ABC, were constructed as independent objects, so the team did not belabor discussing it. The second figure is an isosceles triangle DEF. The lengths of DE and DF are equal. Points D and E are independent objects and point F is constrained to a hidden circle with radius DE.

Figure 2: Constraints of Different Triangles Task
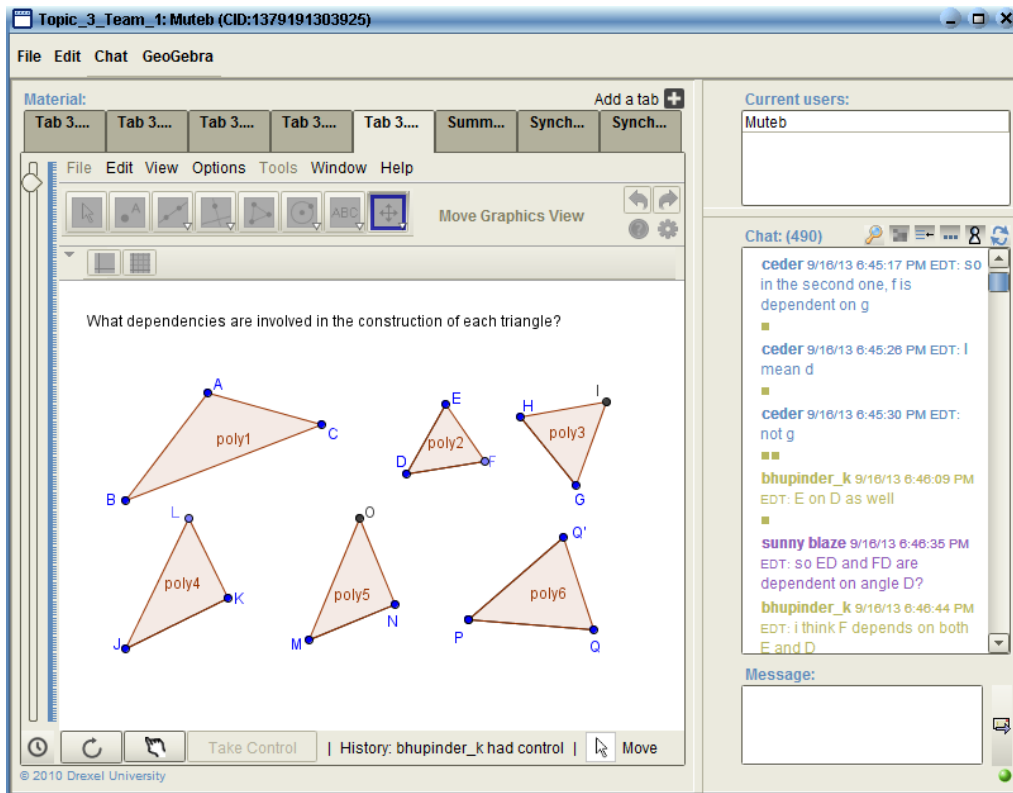
Source: Authors' archive.

The flowing excerpt from Team 1's discussion shows how the team investigated the dependencies of the second figure (poly2):

386 ceder: so in the second one, f is dependent on g

387 ceder: I mean d

388 ceder: not g

389 bhupinder_k: E on D as well

390 sunny blaze: so ED and FD are dependent on angle D?

391 bhupinder_k: i think F depends on both E and D

392 ceder: f doesnt look dependent on anything now...am I imissing something?

393 ceder: ok, what am I missing? F can move independently, but when E is moved, F moves, so that makes which one dependent?

394 bhupinder_k: when you move F, ED stays fixed

395 ceder: right, so F is free to move anywhere

396 ceder: but not when E is moved

397 ceder so F is sometimes dependent?

The team discusses dependencies among points, segments, and angles. In lines 386 to 388, ceder states that F is dependent on D then dismisses her assertion in line 392. Then, sunny blaze states her understanding in a form of questions: "so ED and FD are dependent on angle D?" (line 390). This highlights the struggle the teachers had to identify the dependency when the points are partially dependent. At line 397, ceder asks whether point "F is sometimes dependent". Though they had already seen and, a week before, constructed dependent objects in their first collaborative session, they struggled with a new and more complex situation. The concept of dependency is key for developing utilization schemes that allow learners to identify and build relationships in geometric constructions.

In a latter task, the team uses the concept of dependency to identify relations among objects. The task presents two circles constructed using the same radius, AB. Their points of intersections, C and D, were connected to create a perpendicular bisector to radius AB. The perpendicular bisector CD intersects with AB at point E. The following excerpt shows parts of the team's discussion of the relationships in the figure.

197 ceder: looks like C, D, E are all dependent on A and B

198 bhupinder_k: right

199 sunny blaze: so i notice that as I drag A circle B changes, so circle B depends on A. and vice versa. since they both share the same radius, their areas are equal

200 bhupinder_k: CB = AC=AD=BD

201 ceder: yeah, I agree that the area of the circles depends on the line segment AB

202 sunny blaze: CE=DE and BE=AE

203 ceder: right

204 ceder: I think that covers the line segments

205 ceder: CD is dependent on AB

206 sunny blaze: is this the perpendicular bisector thing (not sure what it's called)

207 bhupinder_k: question: the points that are black are always gonna be dependent?

208 ceder: that's what I have noticed

In the excerpt, ceder states that points C, D, and E are dependent on A and B (line 197). The other teachers agree with her. In line 199, sunny blaze states after dragging the figure, the two circles share the same radius and that dragging the center of one circle affects the size of the other, which makes the circles dependent on the centers. In lines 200-204, the teachers agree on the relationships they notice about the circles. Then ceder states the dependency between line segments CD and AB. The discussion of dependencies helps bhupinder_k to notice the different colors of dependent points. The team successfully reconstructed the figure. Through co-action, teachers appropriated the concept of dependency and used it to understand constructions.

Team 3:

For the purpose of this report, we present the work of this team in two tasks: constructing an equilateral triangle and constructing a perpendicular line that passes through an arbitrary point. This team of teachers worked on appropriating the dragging affordance of VMTwG in the first session, which was evident in the second session when they were constructing perpendicular line that passes through an arbitrary point.

The first task asks teachers to drag an equilateral triangle whose vertices are two centers

of two congruent circles that shares the same radius and their intersection point. The task then asks the teachers to discuss what they notice about the given figure and then construct a similar one in GeoGebra. Before this session, the teachers were asked to drag and notice relationships among basic geometrical objects to become aware of co-active relations between their actions and reactions of the VMTwG environment. As the following excerpt from Team 3's chat log shows, the teachers, gouri and sophiak, felt the necessity to revisit their understanding of dragging after being instructed to create an equilateral triangle.

26   sophiak: It seems that point C is fixed but pts A&B are not. I am thinking somehow A&B were used to create the circles which is why the make the circles bigger or smaller.
27   sophiak: How about you try to explore now?
28   gouri: ok I'll continue on with #2 [the second instruction in Task 8] as well
29   sophiak: No, I would like to create the objects as well. I think it is valuable if we both explore
30   gouri: C does seem fixed/constrained
31   gouri: sure - how about i do it and then you do it as well after?
32   sophiak: Sounds good. Please type what you do.
33   gouri: So far I created 2 circles
34   gouri: and overlapped the D point as the raius point for E
35   gouri: one more try
36   gouri: ok - i deleted the other circle because i dont need it
37   gouri: I somehow thought i could create all 3 points, abc through two circles
38   sophiak: How did you create F?
39   gouri: I added a point
40   gouri: then the polygon tool for the triangle
41   sophiak: Did you want to explore your picture to see if it behaves the same way as the original?
42   gouri: ok
43   gouri: [after dragging the pre-constructed figure for few minutes] I noticed that it's the points that make the circle dynamic

44   gouri: and not the circle (in black) itself

The teachers started by stating their noticings of the construction. In line 26, sophiak mentions that point C is fixed (intersection point of the two circles) and points A and B are not. She also states that points A and B are used to construct the two circles since dragging points A and B affects the two circles. It indicates how sophiak views the relationship between dependency and construction and how she is working on identifying the hotspots of the figure. The second team member successfully creates a similar figure to the task's figure. She states after dragging in lines 43 and 44 that "the points that make the circle dynamic… not the circle (in black) itself". These comments suggest that gouri was concerned with hotspots in VMTwG.

A concern with hotspots is significant to understand the dragging affordance of DGEs. Teachers' understanding of dragging different types of objects, hotspots and other objects, in DGE helped them appropriate the environment, which influenced the type of knowledge that teachers developed later in the course. In this event, the co-active relation between the teachers and the environment helped the teachers develop an understanding of dragging in DGE. This shows how teachers appropriate the environment through developing their understanding of dragging and dependencies. Their work on subsequent tasks illustrates this. In latter sessions in the course, the teachers constructed a line perpendicular to a give line (Task 20), but were unable to construct a line perpendicular to a give line that passes through an arbitrary point (Task 21). Task 20 presents hints for the learners to construct more objects and notice different behaviors (see Figure 3).

In general, our tasks include hidden hints and challenges, accessible through checkboxes, which teams can reveal at any moment. These hints and challenges are intended to keep team members engaged by suggesting actions they can perform and to support productive discursive interactions among team members by inviting them to notice and discuss particular objects and issues (POWELL; ALQAHTANI, 2015). While working on Task 20, Team 3 members were able to construct a valid figure and decided not to check the hints. However, while they were struggling with Task 21, they revisited Task 20 and revealed its hints looking for ideas that could help them solve Task 21.

Figure 3: Task 20: Constructing the midpoint of a line segment

Source: Authors' archive.

The teachers met again and successfully solved Task 21. They used some insights from Task 20 to construct perpendicular lines multiple times. They started by constructing a line AB and an arbitrary point C (see Figure 1). Then using the technique from Task 20 (constructing circles with a common radius, mark their intersection points, then connect them), they constructed a line EF perpendicular line to AB and dragged points A and B to test the construction. On that line, they marked point G and, employing the Task 20 technique, used it and point E to construct line IJ perpendicular to EF, which make IJ parallel to AB. After that, they construct circle EC and marked the intersection point of this circle with line IJ, point K. They dragged point C to test the behavior of the construction. Finally, they construct line KC, which is perpendicular to AB and passes through the arbitrary point C.

Proving that KC is perpendicular to AB is beyond the scope of this paper; however, it can be done using triangle congruency. The teachers collectively constructed their final solution. After each step of their construction, they dragged points A, B, and C to make sure that at each stage their construction maintained properties they intended. Their appropriation of dragging–what to drag, how to drag, and what to expect–was dominant in their problem solving of Task 21.

Discussion

We introduced teams of teachers to a collaborative, online, dynamic geometry environment, VMTwG, in a 15-week professional development course. They interacted to manipulate and construct geometric figures and identify dependencies among geometrical objects. To identify dependencies among geometrical objects in DGEs, users manipulate geometric figures and attend to variances and invariances of the objects that comprise the figures. Our analysis of teachers' interaction in two iterations of the course allowed us to understand how they appropriate collaboratively the tools of VMTwG (instrumentation process) and how their appropriation shapes their geometrical knowledge. Team 1 worked on understanding dependencies among geometrical objects through dragging. Team 3 paid special attention to the characteristics of the objects they dragged. Their

interactions indicate that they perceived the significance of dragging hotspots (points involved in the construction) of a figure (HEGEDUS; MORENO-ARMELLA, 2010), which can be seen as a sign of the development of their schemes. They distinguished between dragging different types of objects. Co-action helped both teams identify hotspots and use them to test their constructions and become aware of dependencies.

Team 1 attended to the reactions of the environment to their actions, which enabled their instrumentation process to occur and support their understanding of the concept of dependency. Engaging with tasks where dependencies are key relations among geometrical objects was an important step. These tasks triggered a discussion about how to use the notion of dependency to create valid constructions. The teachers' discussion was an important step that enabled them to understand how to apply their new concept, dependency. Next, they tested their understanding with another construction. After developing and testing their understanding of dependency, they applied their understanding in another task and identified dependencies among new sets of geometrical objects. Focusing on the logical relations among the geometrical objects (HEWITT, 1999; POWELL; ALQAHTANI, 2015) allowed the teachers to use the environment's tools to examine carefully the dependencies involved in the construction

Teachers' appropriation of dragging and understanding of dependencies among geometrical objects were evident in their subsequent problem solving. Team 3 referred to a previous task (Task 20) multiple times and collectively constructed their solution for constructing a perpendicular line that passes through an arbitrary point (Task 21). They dragged the hotspots of their construction after each step. Their understanding of dragging hotspots helped them solve the task. Interestingly, to construct a perpendicular line that passes through an arbitrary point, the teachers constructed a rectangle. Their procedure anticipated work they would do in a later task.

This study offers two contributions to the literature on teaching and learning mathematics with dynamic geometry environments. Our first contribution concerns the role of dragging in DGEs, which has been the focus of research on how DGEs support learning of mathematics (ARZARELLO ET AL., 2002; BACCAGLINI-FRANK; MARIOTTI, 2010; FALCADE et al., 2007; HOLLEBRANDS, 2007; TALMON; YERUSHALMY, 2004). These studies contributed understanding about dragging modalities and learners' understanding of dependency and covariation. Our study, using the lens of instrumental genesis, contribute understanding of how learners collaboratively appropriate DGEs. Specifically, learners' instrumental genesis of DGEs and understanding of dynamic geometry depends on their appropriation of the dragging affordance. Appropriating dragging influenced teachers' actions in VMTwG and their mathematical discussions while working on geometrical tasks.

The second contribution of this study concerns the design of the VMTwG environment. Four design features of the environment–online setting, collaboration, coherent sequence of tasks, and the independence of learners in problem-solving sessions (instructor is not present)–promote productive mathematical discourse. Solving open-ended geometry problems in a setting that integrated these four features allowed teachers to appropriate the environment and advance their geometrical knowledge. A significant feature of tasks in such environment focuses on mathematical relations and takes advantage of technological tools. Working on such tasks, learners have opportunities to manipulate mathematical objects and discuss their observations to identify and justify mathematical relations among objects.

Even though dragging is considered the main affordance in DGEs, other tools play important role during problem-solving activity in DGEs. Further research is needed to understand how appropriating other tools of DGEs occurs and how this appropriation influences mathematical knowledge. In this study, the collaborative aspect of the environment influenced teachers' mathematical activity, which calls for deeper investigation of how collaboration supports leaners' appropriation of DGEs and their mathematical understanding. In addition, research is needed to investigate how teachers' understanding of DGEs shapes how they integrate them into their teaching practice.

Funding

Note

1 Parts of this work were presented at the 39th annual conference of the International Group for the Psychology of Mathematics Education in Hobart, Australia.

References

ALQAHTANI, M. M.; POWELL, A. B. Teachers' instrumental genesis and their geometrical understanding in a dynamic geometry environment. *Digital Experiences in Mathematics Education, 3*(1), 2017, p. 9-38. doi:10.1007/s40751-016-0025-5

ARZARELLO, F.; OLIVERO, F.; PAOLA, D.;ROBUTTI, O. A cognitive analysis of dragging practises in Cabri environments. *ZDM Mathematics Education, 34*(3), 2002, p. 66-72.

BACCAGLINI-FRANK, A.; MARIOTTI, M. A. Generating conjectures in dynamic geometry: The maintaining dragging model. *International Journal for Computers in Mathematical Learning, 2010, 15, p.* 225–253.

CHEUNG, A. C.; SLAVIN, R. E. The effectiveness of educational technology applications for enhancing mathematics achievement in K-12 classrooms: A meta-analysis. *Educational Research Review, 9*, 2013, p.88-113.

FALCADE, R.; LABORDE, C.; MARIOTTI, M. A.. Approaching functions: Cabri tools as instruments of semiotic mediation. *Educational Studies in Mathematics, 66*(3), 2007, p. 317-333.

GATTEGNO, C. *The science of education:* Part 1: Theoretical considerations. New York: Educational Solutions, 1987.

GOLDENBERG, E. P. Mathematics, metaphors, and human factors: Mathematical, technical, and pedagogical challenges in the educational use of graphical representation of functions. *The Journal of Mathematical Behavior, 7*, 1988, p. 135-173.

GUIN, D.; TROUCHE, L. The complex process of converting tools into mathematical instruments: The case of calculators. *International Journal of Computers for Mathematical Learning, 3*(3), 1998, p. 195-227.

GUVEN, B.; CEKMEZ, E.; & KARATAS, I. Using

empirical evidence in the process of proving: the case of Dynamic Geometry. *Teaching Mathematics and its Applications, 29*(4), 2010, p. 193-207.

HEGEDUS, S. J.; MORENO-ARMELLA, L.. Accommodating the instrumental genesis framework within dynamic technological environments. *For the Learning of Mathematics, 30*(1), 2010, p. 26-31.

HEWITT, D. Arbitrary and necessary part 1: A way of viewing the mathematics curriculum. *For the Learning of Mathematics, 19*(3), 1999, p.2-9.

HIGGINS, S.; XIAO, Z.; KATSIPATAKI, M. The impact of digital technology on learning: A summary for the education endowment foundation. Durham, UK: *Education Endowment Foundation and Durham University, 2012.*

HOHENWARTER, J.; HOHENWARTER, M.; LAVICZA, Z. Introducing dynamic mathematics software to secondary school teachers: The case of GeoGebra. *Journal of Computers in Mathematics and Science Teaching, 28*(2), 2009, p. 135-146.

HOLLEBRANDS, K. F. The role of a dynamic software program for geometry in the strategies high school mathematics students employ. *Journal for Research in Mathematics Education, 38*(2), 2007, p. 164-192.

HSIEH, H.-F.; SHANNON, S. F. Three approaches to qualitative content analysis. *Qualitative Health Research, 15*(9), 2005, p. 1277-1288.

KAPUT, J. J.; THOMPSON, P. W. Technology in mathematics education research: The first 25 years in the JRME. *Journal for Research in Mathematics Education, 25*(6), 1994, p. 676-684.

LABORDE, C. Core geometrical knowledge for using the modelling power of Geometry with Cabri-geometry. *Teaching Mathematics and its Applications, 18*(4), 1999, p. 166-171.

LI, Q.; MA, X. A meta-analysis of the effects of computer technology on school students' mathematics learning. *Educational Psychology Review, 22*(3), 2010, p. 215-243.
MORENO-ARMELLA, L.; HEGEDUS, S. J. Co-action with digital technologies. *ZDM Mathematics Education, 41*(4), 2009, p. 505-519. doi:10.1007/s11858-009-0200-x

NATIONAL GOVERNORS ASSOCIATION

CENTER FOR BEST PRACTICES; COUNCIL OF CHIEF STATE SCHOOL OFFICERS. *Common core state standards for mathematics*, 2010. Retrieved from http://www.corestandards.org/wp-content/uploads/Math_Standards1.pdf. Access in March 10, 2018.

ONER, D. Supporting students' participation in authentic proof activities in computer supported collaborative learning (CSCL) environments. *Computer-Supported Collaborative Learning, 3*, 2008, p. 343-359.

PIEZ, C. M.; VOXMAN, M. H. Multiple representations—Using different perspectives to form a clearer picture. *The Mathematics Teacher*, 1997, p.164-166.

PIMM, D. *Speaking mathematically: Communication in mathematics classrooms*: Routledge & Kegan Paul London, 1987.

POWELL, A. B.; ALQAHTANI, M. M.. Tasks and meta-tasks to promote productive mathematical discourse in collaborative digital environments. In I. Sahin, A. Kiray, & S. Alan (Eds.), *Proceedings of the International Conference on Education in Mathematics, Science & Technology*. Antalya, Turkey, 2015, p. 84-94.

POWELL, A. B.; GRISI-DICKER, L. *Toward collaborative, discourse-focused learning with dynamic geometry environments*. Paper presented at the 12th International Congress on Mathematical Education, Seoul, Korea, 2012.

RABARDEL, P.; BEGUIN, P. Instrument mediated activity: from subject development to anthropocentric design. *Theoretical Issues in Ergonomics Science, 6*(5), 2005, p. 429-461.

RAKES, C. R.; VALENTINE, J. C.; MCGATHA, M. B.; RONAU, R. N.. Methods of Instructional Improvement in Algebra A Systematic Review and Meta-Analysis. *Review of Educational Research, 80*(3), 2010, p. 372-400.

SAMPER, C.; CAMARGO, L.; MOLINA, Ó.; PERRY, P.. Instrumented activity and semiotic mediation: two frames to describe the conjecture construction process as curricular organizer. In A. Lindmeier & A. Heinze (Eds.), *Proceedings of the 37th Conference of the International Group for the Psychology of Mathematics Education*. Kiel, Germany: PME, 2013, Vol. 4, p. 145-152).

SPRINGER, L.; STANNE, M. E.; DONOVAN, S. S. Effects of small-group learning on undergraduates in science, mathematics, engineering, and technology: A meta-analysis. *Review of Educational Research,* 1999, 69(1), p. 21-51.

STAHL, G. *Group cognition:* Computer support for building collaborative knowledg*e*. Cambridge, MA: MIT, 2006.

STAHL, G. *Constructing dynamic triangles together:* the development of mathematical group cognition. UK: Cambridge University Press, 2015.

TALMON, V.; YERUSHALMY, M. Understanding dynamic behavior: Parent–Child relations in dynamic geometry environments. *Educational Studies in Mathematics, 57*(1), 2004, p.91-119.

WEBB, N. M.; PALINCSAR, A. S. Group processes in the classroom. In D. C. Berliner & R. C. Calfee (Eds.), *Handbook of educational psychology*. New York, NY: Macmillan, 1996, p. 841-873 .

WENGLINSKY, H.. *Does it compute? The relationship between educational technology and student achievement in mathematics*. 1998. Retrieved from http://www.ets.org/Media/Research/pdf/PICTECHNOLOG.pdf. Access in March 10, 2018.

About authors

Muteb M. Alqahtani – State University of New York at Cortland. Ph.D. in mathematics education from Rutgers University-New Brunswick. Assistant professor of mathematics education, Childhood/Early Childhood Education Department, State University of New York at Cortland.

Arthur B. Powell – Rutgers University-Newark in New Jersey. Professor of mathematics education in the Department of Urban Education and Faculty Research Scientist and Associate Director of the Robert B. Davis Institute for Learning of the Graduate School of Education, Rutgers University-New Brunswick. Editor of *Métodos de pesquisa em educação matemática—Usando escrita, vídeo e internet*, Mercado de Letras, 2015.